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Final Report – Phase I Project

Michigan Engineering Services, LLC
2890 Carpenter Road, Suite 1900
Ann Arbor, MI 48108

Principal Investigator:
Nick Vlahopoulos, Chief Technical Officer, 734-358-0792, e-mail: nv@miengsrv.com

Corporate Official:
Christina Vlahopoulos, President, 734-358-0777, e-mail: christinavl@miengsrv.com

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Government Technical Monitor: Ms. Kelly Cooper, ONR Code 331

Phase I Acquisition Sponsor: DD(X)

Research Institution: Department of Naval Architecture and Marine Engineering
University of Michigan
PI: David Singer

Table of Contents

Nomenclature

HLS	Heavy Lift Ship
FEA	Finite Element Analysis
FRM	Fast Running Model
MDO-U	Multidisciplinary Design Optimization under Uncertainty
MES	Michigan Engineering Services, LLC
MPP	Most Probable Point
ND	Non-Dimensional
PCD	Principal Component Decomposition
RBDO	Reliability Based Design Optimization
SM	Section Modulus
SPP	Seakeeping Prediction Program
SS-MDO-U	System-of-Systems Multidisciplinary Design Optimization under Uncertainty
TC	Target Cascading
TS	Transport Ship
UM	University of Michigan
w.r.t.	with respect to

I. Project Summary

   All objectives that were stated in the Phase I proposal were met or exceeded by the Phase I effort. Specifically, the following work was completed:

(i)   Development of an operational SS-MDO-U system.
(ii)   Introduction of FRM technology in the SS-MDO-U system.
(iii)   Validation and case – study of the SS-MDO-U development. A simple example with anticipated results was utilized initially for the validation. A case study for the simultaneous design of two ships which are expected to operate and function together were analyzed (Figure 1). Multiple disciplines were considered during the design of each ship (system level optimization) and the combined performance of the two ships demonstrated the system-of-systems functionality. This is a feasibility demonstration of how the SS-MDO-U system will be applied for designing new complex concepts such as the seabasing, after the SS-MDO-U capability is fully developed.
(iv)   Roadmaps for future work and the development of a commercial transition plan were completed.
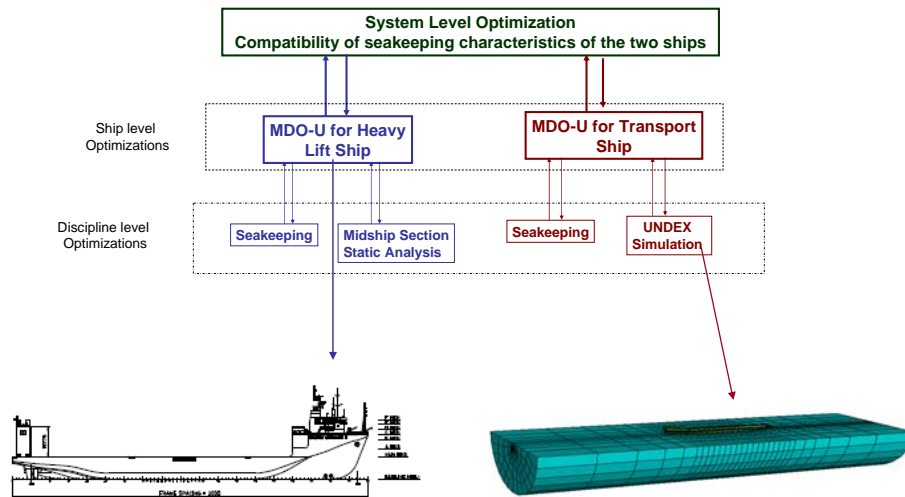
Figure 1. Flow Chart of the Case Study completed during the Phase I effort

The SS-MDO-U process developed by MES allows associating performance requirements and economic metrics with available performance assessment simulation tools (high or low fidelity). This is an empowering new capability because it organizes the complex decision making process and allows to identify how requirements on individual disciplines or systems interact with each other and how they impact the overall cost. If this is not done properly during a complex designing process then the cost can become prohibiting.

   In this report technical information is presented about the completed tasks. The roadmap for future work and the commercial development plan are included in the submitted Transition STTR Plan. Overall this development sets the technical foundation for developing a computational tool for assisting Naval engineers during the decision making process when designing a system of multiple ships which must interact and operate jointly. The fleet of ships comprises the system-of-systems, and each ship comprises each system. Multiple disciplines are considered for each ship during the design process.

II. Development of the SS-MDO-U System

A capability was developed along the lines of the flow chart presented in Figure 2. It is based on combining multiple MDO-U blocks together for performing a system-of-systems level analysis. The flow chart for each MDO-U block is presented in Figure 3; it optimizes each separate system by considering the multiple disciplines associated with the performance of each system. The MDO-U optimization capability has been developed by MES under current Phase II SBIR funding from NASA Ames. Previously the MDO-U system was utilized for entry vehicle design by combining multiple disciplines (trajectory, aerodynamics, guidance and control, aerothermal, and thermal analyses), while considering uncertainties in the thermal loads and in the properties of the thermal protection systems. In this project the MDO-U capability comprised the building block for generating the SS-MDO-U system.

Figure 2. Flow Chart of SS-MDO-U process

The TC method is employed in this work at two levels; for organizing the multiple optimizations for each discipline within each MDO-U block; and on a system-of-systems level for organizing the optimizations of the multiple systems. The flow chart for the MDO-U process is presented in Figure 3.
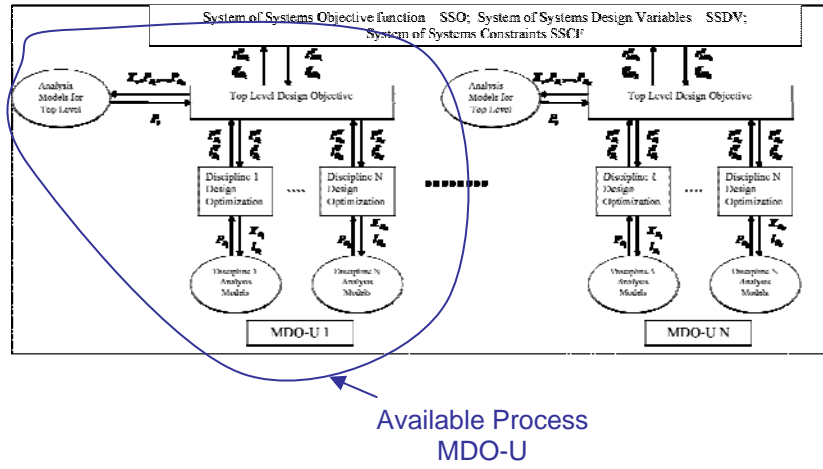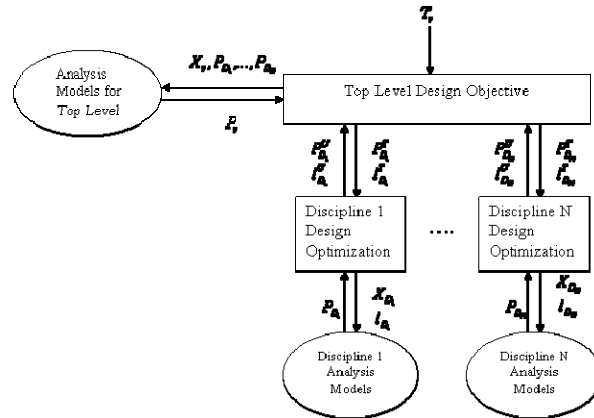

Figure 3. Representative flow chart of the Target Cascading method

3

At the top level of the optimization process in the TC method lays the optimization statement which defines the overall objective for the particular ship system. The top level optimization, tries to make the system's performance represented by the P_v response to match the targeted performance defined as T_v by the user. The analysis model takes into account the variables describing the performance of the system $((P_{D_1},...,P_{D_N})$ in each one of the N disciplines $(D_1,...,D_N)$ which are considered. The analysis model also takes into account the variables X_v which define the design characteristics of the system. The system model constitutes a mathematical statement which assesses how the design variables and the performance from each discipline are combined in order to evaluate the overall system performance. For example, the system model can be a cost model which assesses the value of a system based on the overall life cycle cost. In this manner, when the expectations for the performance from each discipline are defined by the user, the corresponding impact on the overall cost can be identified. Within each iteration of the top level design optimization process, a separate optimization problem is solved for each discipline. The linking design variables $l_{D_i}^T$ are passed from the top "T" level optimization, to the optimization problem at each discipline level. The performance of each discipline desired by the top optimization problem is also defined as $P_{D_i}^T$.

At each discipline level a design optimization problem is solved using the appropriate analysis model. Within each iteration of the discipline level optimization problem, the design variables X_{D_i} and the linking variables l_{D_i} are provided to the simulation model. The predicted performance P_{D_i} is returned from the simulation model. Once the discipline level optimization analysis is completed and the optimal solution is reached, the updated "U" values for the linking variables $l_{D_i}^U$ and the updated performance $P_{D_i}^U$ are returned to the top level optimization problem. Then, the next iteration in the top level optimization problem is performed. The mathematical statement for the top level optimization problem is:

$$
\min_{X_v, P_{D_i}, l_{D_i}} \left\| P_v - T_v \right\| + \varepsilon_v^P + \varepsilon_v^l
$$

subject to: $\displaystyle\sum_{i=1}^{N} \left\| P_{D_i} - P_{D_i}^U \right\| \le \varepsilon_v^P$, $\displaystyle\sum_{i=1}^{N} \left\| l_{D_i} - l_{D_i}^U \right\| \le \varepsilon_v^l$, $g_v(P_{V_i}, X_v, l_{D_i}) \le 0$, $h_v(P_{V_i}, X_v, l_{D_i}) = 0$ (1)

where g_v presents the inequality constraints and h_v presents the equality constraints of the top level design optimization problem. The mathematical statement for each discipline level design optimization problem is:

$$
\min_{X_{D_i}, l_{D_i}} \left\| P_{D_i} - P_{D_i}^T \right\|
$$

subject to: $g_{D_i}(P_{D_i}, X_{D_i}, l_{D_i}) \le 0$, $h_{D_i}(P_{D_i}, X_{D_i}, l_{D_i}) = 0$ (2)

The overall design optimization objective is achieved by letting the top level optimization (which is based on the performance of each discipline) to exchange information with each discipline in a very systematic manner. Different disciplines can share the same variables since

elements of the l_{D_i} groups of linking variables can be the same for different groups. Thus, the inter-relationships between disciplines are accounted, while retaining a very structured and efficient manner for optimizing the design. Another major benefit of this method is that the user can employ different optimization schemes for each discipline.

In order to develop the SS-MDO-U process, a higher level driver is placed on top of the current MDO-U process by repeating the Target Cascading approach at a higher level. Figure 2 presents details for the new SS-MDO-U development completed in this project in a flow chart form. Each MDO-U block addresses the performance of each ship, and the higher level optimization process interconnects the design process of each system by requiring for the various ship systems to operate within a fleet of ships. For example, the seabasing concept represents a situation where the new developments can be utilized. Linking variables l_{MD_i} and performance metrics P_{MD_i} are passed between the higher level and the systems' level design processes. At the higher system of systems level, the objective function, the design variables, and the constraints are considered along with augmentations in the objective function introduced by the limits of the constraints imposed by the TC method. The mathematical statement for the higher level (system of systems) design problem is:

$$\min_{SSDV, P_{MD_i}, l_{MD_i}} \quad SSO + \varepsilon^P_{MDO-U} + \varepsilon^l_{MDO-U} \tag{3}$$

subject to:

$$g_{SS}(SSDV, P_{MD_i}, l_{MD_i}) \leq 0, \quad h_{SS}(SSDV, P_{MD_i}, l_{MD_i}) = 0, \quad \sum_{i=1}^{N} \left\| P_{MD_i} - P^U_{MD_i} \right\| \leq \varepsilon^P_{MDO-U} \ ,$$

$$\sum_{i=1}^{N} \left\| l_{MD_i} - l^U_{MD_i} \right\| \leq \varepsilon^l_{MDO-U} \tag{4}$$

where SSO is the higher level objective function, $SSDV$ are the higher level design variables, g_{ss} and h_{ss} are the inequality and equality constraints, P_{MD_i} and l_{MD_i} are the performance functions and the linking variables of each system level MDO-U process and superscript $"U"$ indicates the values returned from each system level design to the higher level system of systems design driver.

Uncertainty is introduced in the SS-MDO-U process through a single – loop RBDO algorithm for incorporating uncertainty in the TC method. The single-loop RBDO method imposes the optimality conditions of the reliability loops as an equivalent deterministic equality constraint in the design optimization loop. It therefore, converts the probabilistic optimization formulation into an equivalent deterministic optimization formulation. It does not need to calculate the MPP for each probabilistic constraint in order to assess the reliability, thus eliminating the reliability optimization loop of the conventional double-loop RBDO approach. In general, this single-loop RBDO approach has comparable efficiency with a deterministic optimization and the same accuracy with the conventional double-loop RBDO method. It is however, much more efficient compared with the double-loop and decoupled methods.

Based on the R-percentile formulation, the single-loop RBDO problem is stated by the following equivalent deterministic optimization problem:

$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{x}} f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X},\boldsymbol{\mu}_\mathbf{p})$$

subject to: $\quad G_i(\mathbf{d},\mathbf{X}_i,\mathbf{p}_i) \geq 0 \qquad$ i=1,2,……n $\hfill$ (5)

where

$$\mathbf{X}_i = \boldsymbol{\mu}_\mathbf{X} - \boldsymbol{\sigma} * \beta_{t_i} * \boldsymbol{\alpha}_i$$

$$\mathbf{p_i} = \boldsymbol{\mu_p} - \boldsymbol{\sigma} * \beta_{t_i} * \boldsymbol{\alpha_i} \hfill (6)$$

$$\boldsymbol{\alpha}_i = \boldsymbol{\sigma} * \nabla G_{i\,\mathbf{X,p}}(\mathbf{d},\mathbf{X}_i,\mathbf{p}_i) / \| \boldsymbol{\sigma} * \nabla G_{i\,\mathbf{X,p}}(\mathbf{d},\mathbf{X}_i,\mathbf{p}_i) \|$$

$$\mathbf{d}^l \leq \mathbf{d} \leq \mathbf{d}^u; \quad \boldsymbol{\mu_X}^l \leq \boldsymbol{\mu_X} \leq \boldsymbol{\mu_X}^u .$$

In Eq. (5) $\mathbf{d} \in R^k$ is the vector of deterministic design variables, $\mathbf{X} \in R^m$ is the vector of random design variables, $\mathbf{p} \in R^q$ is the vector of random and deterministic design parameters and *n, k, m* and *q* are the number of constraints, deterministic design variables, random design variables and design parameters, respectively. Also, β_{t_i} is the target reliability index for the ith constraint, $\boldsymbol{\mu_X}, \boldsymbol{\mu_p}$ are the mean values of $\mathbf{X}$ and $\mathbf{p}$, $\boldsymbol{\alpha}_i$ is the normalized gradient of the ith constraint and $\boldsymbol{\sigma}$ is the standard deviation vector of random variables $\mathbf{X}$ and parameters $\mathbf{p}$.

The objective function is evaluated at the mean point $\mathbf{d}, \boldsymbol{\mu_X}, \boldsymbol{\mu_p}$ and the constraints are calculated at the $\mathbf{d},\mathbf{X},\mathbf{p}$ point. The relationship of Eq. (6) is used to evaluate the constraints consistently with the values of the design variables. The single-loop method does not search for the MPP of each constraint at each iteration. Instead, the MPP of the active constraints are correctly identified at the optimum. This dramatically improves the efficiency of the single-loop method without compromising the accuracy.
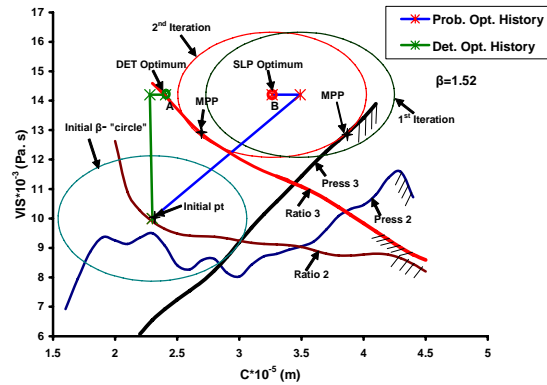
Figure 4. Representative difference between deterministic and probabilistic optimization

The probabilistc optimization problem stated by Eq. (5) and (6) is introduced in the optimization statements of the SS-MDO-U process (Figure 2). This has been accomplished by subdividing the design variables and the linking variables of the SS-MDO-U process into the ones which are considered deterministic and the ones which are considered as random. The constraints of the SS-MDO-U process which contain random design variables or random parameters are modified according to Equations (5) and (6) in order to account for the uncertainties. The modified constraints are essentially pushing the optimal solution further back from the active original constraints of the optimization process. Figure 4 presents graphically the difference between a deterministic and a probabilistic optimum. The probabilistic optimal point

is placed within a safety margin away from the active constraints (e.g. Point B vs. Point A in Figure 4).

During the Phase I STTR effort the framework of the SS-MDO-U process was developed. Although in the original proposal the plan was to just hardwire the SS-MDO-U development in the code, a flexible general purpose SS-MDO-U system was developed instead that allows the user to define all the necessary information for the system-of-systems analysis in a structured data file. This allows for considerably more flexibility when performing the case studies. The functionality of the developed SS-MDO-U system is presented in Figure 5. The user defines in a data file information about the objective functions, the linking variables, and the constraints for the system-of-systems level, the various system levels, and the various discipline levels optimizations. The outline of the structure of the data file is presented in Figure 6.

Operation of SS-MDO-U Process

SS-MDO-U System

Generates optimization script automatically

The following are embedded in the optimization script:
- Target Cascading logic
- Single-loop RBDO
- FRM

Data File

Run optimization script

SS-MDO-U Results
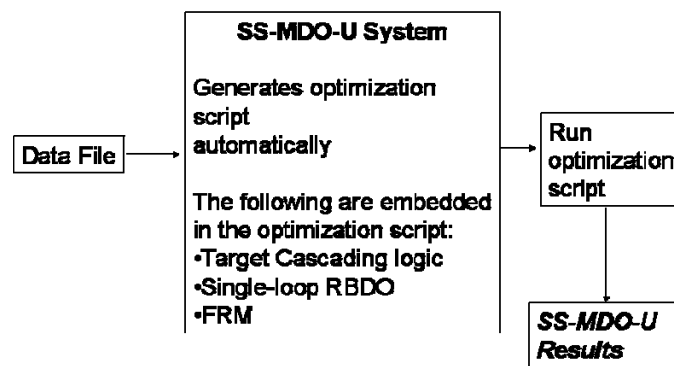
Figure 5. Operational flow chart of the developed SS-MDO-U process

The data files which are utilized in the case study for the two ships system are attached in Appendix A. They contain the definition of the data required to perform the analyses without uncertainty and with uncertainty, respectively. During a potential Phase II effort the SS-MDO-U capability can be developed further for becoming more user friendly and easy to use. Additional technical functionality can also be added to the SS-MDO-U system (all potential new developments are outlined in Section VII).

System of Systems Level Optimization
Design variables, including any uncertainty
Objective function
Constraints
Definition of each System

System level Optimization:
Linking variables
Objective function
Constraints
Definition of disciplines

Discipline level Optimization:
Linking variables
Variables and parameters with uncertainty
Objective function
Utility functions
Constraints
Constraints with uncertainty

Figure 6. Structure of data file for SS-MDO-U process

III. Introduction of FRM Technology in the SS-MDO-U System

Capabilities for developing response surfaces for scalar variables and time dependent metamodels for time dependant responses of interest are integrated in the SS-MDO-U system. The response surfaces for scalar variables are developed using the Kriging method which comprises a Gaussian type of metamodels. When the response of interest is a scalar quantity (i.e. roll, heave, or pitch, etc.) then response surfaces can be created and utilized instead of the actual simulation models during the optimization process. If the response of interest is a time dependent quantity (i.e. time history of stress, velocity time history, etc.) then PCD is combined with the Kriging method for creating the FRM. The FRM are employed during the optimization process instead of the actual solvers for enabling the use of high fidelity models in the design process. The technical background of the Kriging method is presented first. Then, the theory of the FRM is also discussed. Both capabilities have been integrated in the SS-MDO-U system.

The Kriging metamodels are equations that replace an actual simulation model. The equations that define the metamodels are evaluated from actual simulations performed at a limited number of sample points. An optimal symmetric latin hypercube algorithm is used by MES for selecting the sample points. Once the sample points and the values for the performance variables are known, the Kriging method is employed for generating a metamodel for each performance function of interest. The Kriging method is based on treating $Z(\tilde{x})$, the error between the actual performance variable $y(\tilde{x})$ and an approximate value β, as a stochastic process:

$$y(\tilde{x}) = \beta + Z(\tilde{x}) \tag{7}$$

where $\tilde{x}$ is the d-dimensional vector of the variables that defines the point where the performance variable is evaluated, and "d" is the number of variables. $Z(\tilde{x})$ is considered as a normal process with zero mean and a covariance that can be expressed as:

$$\text{cov}(Z(\tilde{x}_i), Z(\tilde{x}_j)) = \sigma^2 R(\tilde{x}_i, \tilde{x}_j) \tag{8}$$

where σ^2 is the process variance and $R(\tilde{x}_i, \tilde{x}_j)$ is the spatial correlation function. The equation used for the spatial correlation function is a Gaussian spatial correlation function:

$$R(\tilde{x}_i, \tilde{x}_j) = \prod_{k=1}^{d} \exp(-\theta_k (x_{i,k} - x_{j,k})^2) \tag{9}$$

and it indicates a process with infinitely differentiable paths in the mean square sense. θ_k is the correlation parameter that corresponds to the kth component of the d-dimensional vector of the random variables $\tilde{x}$, i.e. k=1,2,…,d. For a set $\tilde{x}_s$ comprised by "n" number of sampling points $\tilde{x}_{si}$, i=1,2,…,n

$$\tilde{x}_s^T = \{\tilde{x}_{s1}, \tilde{x}_{s2}, \ldots, \tilde{x}_{sn}\} \tag{10}$$

The corresponding performance variable $\tilde{y}_s$ is considered known and its values are defined as:

$$\tilde{y}_s^T = \{y(\tilde{x}_{s1}), y(\tilde{x}_{s2}), \ldots, y(\tilde{x}_{sn})\} \tag{11}$$

The vector of correlations between the sample points $\tilde{x}_s$ and the evaluation point $\tilde{x}$ can be expressed as:

$$\tilde{r}^T(\tilde{x}) = \{R(\tilde{x}, \tilde{x}_{s1}), R(\tilde{x}, \tilde{x}_{s2}), \ldots, R(\tilde{x}, \tilde{x}_{sn})\} \tag{12}$$

The correlation matrix [R] is also defined among all the sample points:

$$[R] = [R(\tilde{x}_{si}, \tilde{x}_{sj})]_{nxn} \tag{13}$$

The spatial correlation function in Equations (12) and (13) has been defined by Equation (9). In the Kriging method the value of the performance function evaluated by the metamodel at the

evaluation point $\tilde{x}$ is treated as a random variable. The computation of β and $Z(\tilde{x})$ in Equation (7) is based on minimizing the mean square error (MSE) in the response:

$$\text{MSE}\,[\,\hat{y}\,(\tilde{x}\,)] = E\,[\,\hat{y}\,(\tilde{x}\,)\text{-}y\,(\tilde{x}\,)]^{\,2} \tag{14}$$

subjected to the unbiasedness constraint:

$$E\,[\,\hat{y}\,(\tilde{x}\,)] = E[y\,(\tilde{x}\,)] \tag{15}$$

Thus, a set of parameters θ_k (k=1,2,…,d) are computed as the solution of minimizing the product $[(\det(R))^{1/n} \cdot \hat{\sigma}^2]$, where $\hat{\sigma}^2 = \frac{1}{n}(\tilde{y}_s - \tilde{F}\,\hat{\beta})^T\,R^{-1}(\tilde{y}_s - \tilde{F}\,\hat{\beta})$, $\tilde{F}$ is a vector of n-dimension with all unit entries and $\hat{\beta}$ defined as:

$$\hat{\beta} = (\tilde{F}^T\,R^{-1}\,\tilde{F})^{-1}\,\tilde{F}^T\,R^{-1}\,\tilde{y}_s \tag{16}$$

Once the set of optimal parameters θ_k has been computed, then the performance function at any point $\tilde{x}$ can be evaluated as:

$$y\,(\tilde{x}) = \hat{\beta} + \tilde{r}^T(\tilde{x})\,R^{-1}(\tilde{y}_s - \tilde{F}\,\hat{\beta}) \tag{17}$$

where $\tilde{r}^T(\tilde{x})$ is defined in Equation (12). Equation (17) comprises the metamodel developed by the Kriging method.

   The FRM are developed by combining PCD with metamodeling. Once they are created, the FRM replace time consuming simulations when computing the time history of various responses of interest, and provide a very efficient way for performing the simulations. In order to develop the FRM, the time histories for the responses of interest from each sample point are placed in a response matrix [X]. The principal component analysis is performed for each response matrix. Once the principal components have been evaluated for each sample point, the Kriging method is used for generating the metamodels for the principal components. A response matrix is comprised by the time histories of the variables of interest:

$$[X] = \begin{bmatrix} x_1(t_1) & \dots & x_1(t_n) \\ \vdots & \ddots & \vdots \\ x_m(t_1) & \dots & x_m(t_n) \end{bmatrix} \tag{18}$$

Each row in [X] corresponds to a different location within the system which is analyzed, and each column corresponds to a different time step. The PCD allows converting the time histories into "modal" type of information through singular value decomposition. The time histories in the response matrix include all non-linear effects. Through singular value decomposition the response matrix [X] is written as:

$$[X] = [U]\,[W]\,[V]^T \tag{19}$$

where [U] is a matrix which contains "modal" type of information in each column. [W] is a diagonal matrix which contains information about the energy associated with each mode placed in each column of [U]. $[V]^T$ is a matrix which contains time domain information about how the "modes" are synthesized in order to produce the time histories. The PCD provides a compact representation of the response for a non-linear model, and offers an expansion basis for expressing the response of the system. In order to perform the principal component decomposition, equation (19) is re-written as:

$$[X] = \begin{bmatrix} \Phi & \Phi_\tau \end{bmatrix} \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta \\ \eta_\tau \end{bmatrix} \qquad \text{where } [W] = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix},\ [U] = \begin{bmatrix} \Phi & \Phi_\tau \end{bmatrix},\ [V]^T = \begin{bmatrix} \eta \\ \eta_\tau \end{bmatrix} \tag{20}$$

The principal components decomposition is based on equation (20) and results in the following expression for [X]:

$$[X] = [\Phi][D][\eta] \tag{21}$$

The matrices $[\Phi_\tau]$ and $[\eta_\tau]$ do not participate in the definition of matrix [X] since they are both multiplied with the zero entries of matrix [W], and therefore do not contribute in the definition of the principal components. The principal components of the response matrix are employed for developing FRM. Metamodels are developed for the principal components of the response matrix. The Kriging method is used for generating metamodels for the entries of the principal component matrices resulting into the equation for the FRM:

$$[X(\gamma)] = [\Phi(\gamma)][D(\gamma)][\eta(\gamma)] \tag{22}$$

where γ represents the set of parameters which are considered to vary during the design analysis. The FRM are utilized for including high fidelity simulations in the SS-MDO-U process.

IV. Simple Validation Example

In order to demonstrate that the SS-MDO-U system performs as anticipated a simple example is analyzed first. Two identical single degree of freedom systems are analyzed. Three disciplines are considered for each single degree of freedom system; a static analysis, a normal mode dynamic analysis, and a forced frequency response. Each single degree of freedom system comprises a system level optimization. At the system-of-systems level the two systems are considered to be connected in series and their combined static response comprises the response function of interest. The two systems are selected to be identical at the starting point of the design in order to test that the final configurations identified by the SS-MDO-U process are the same for both systems, due to mathematical symmetry.

Figure 7 presents the definition of the two system level optimization including a summary of the analyses, the top level optimization problem, and the optimization problems which are considered at each discipline level. It also depicts the optimization statement at the system-of-systems level. This simple system is selected to be analyzed because its physics offer a challenge to the optimization solution due to competing directions and the limiting constraints, but yet it is feasible to check the optimization results due to the simplicity of the system. Uncertainty is considered in the stiffness property of the system. This uncertainty impacts the solution due to the constraint which is imposed on the maximum allowed natural frequency on each one of the two systems.

The top optimization problem at each system has as an objective to minimize the mass M of the system, while at the same time optimizing the objectives associated with each one of the discipline level analyses. A lower limit $M_{low} = 1$ is imposed on how small M can become. The first discipline has as an objective to minimize the magnitude of the static displacement under a prescribed static load F_{st}. This requirement implies an expected increase in the stiffness of the system K.

The second discipline had as an objective to maximize the natural frequency of the system (i.e. $\omega_o = \sqrt{\dfrac{K}{M}}$) but with an upper bound imposed on the natural frequency $\omega_o \le \omega_{max}$. This optimization problem implies an expected increase in K and a decrease in M. However, due to the constraint $\omega_o \le \omega_{max}$, it is expected that a limitation will be imposed on how much K will be

increased and how much M will be reduced. An uncertainty is introduced in the value of the stiffness K, which is considered as a random variable with prescribed standard deviation. Due to the variability in K, it is expected that the mass M will be allowed to be reduced less than before in order to still satisfy the constraint $\omega_o \le \omega_{max}$ in the presence of uncertainty. The variation in K will make ω_o take larger values than expected when K acquires larger values. In order to compensate for this it is expected that M will not be allowed to be reduced as much as before.

The third optimization discipline is associated with the forced frequency response analysis of the system. A dynamic load $F\,e^{j\omega t}$ is applied on the system and the absolute value of the amplitude is minimized (i.e. $\left|\dfrac{F}{-\omega^2 M + K}\right|$). This discipline can drive the solution towards two possible directions. Either M will be reduced and K will be increased, or M will increase and K will be reduced. Considering that the overall objective is to reduce M, and that the objectives of the first two disciplines are met only with an increase in K and a reduction in M, it is expected that the solution in the third optimization discipline will be guided towards the former path. The system-of-systems level optimization statement is minimizing the static response of the system of the two springs connected together (i.e. $F_{st}((1/K_1) + (1/K_2))$)

In the application of the SS-MDO-U code for solving this problem the following values are used for defining the optimization problem: $K = 100\,N/m$ and $M = 10\,kg$ comprise the starting point for the optimization; $F_{st} = 2{,}000\,N$ (at both the system-of-systems level, and at the two static analysis disciplines), $\omega_{max} = 42$ is the limit of the constraint, and $F\,e^{j\omega t} = 200\,e^{j\,9\,t}$ is the dynamic excitation. The results from the analysis without considering uncertainty and the ones with uncertainty are summarized in Table 1. The results for both systems are identical as expected. In the analysis with uncertainty the standard deviation for the stiffness is considered to be $\sigma = 100$N/m and the required reliability level is set to 98%.

	initial value	final value after optimization without uncertainty	final value after optimization with uncertainty
System-of-Systems Level			
X_{st}	40	2.269	2.276
System level (the results for both Systems are identical)			
K	100	1,762.64	1,757.39
M	10	1	1.113
X_{st}	20	1.134	1.1381
ω	3.1623	41.98	39.73
X_{dyn}	0.2817	0.1189	0.12

Table 1. Summary for SS-MDO-U results from analyses without and with uncertainty

As expected, when uncertainty is considered in the calculations, the amount of reduction in the masses of the systems is less and the increase in the stiffnesses is not as much, leading to a more conservative design that does not achieve the same level of optimization as the solution without uncertainty, but satisfies the constraints in the presence of uncertainty. When the stiffness

acquires its mean value, the natural frequency acquires the value of $\omega_o = 39.73$ Hz. Thus, a safety margin is introduced between the limiting value of 42Hz and the value of ω_o. The safety margin depends on the prescribed level of reliability for meeting the constraint. In the presence of uncertainty the solution without uncertainty provides a natural frequency equal to $\omega_o = 44.37$ (i.e. it violates the constraint), while the solution with uncertainty brings the value of $\omega_o = 42$ (i.e. right at the bounds of the probabilistic constraint).
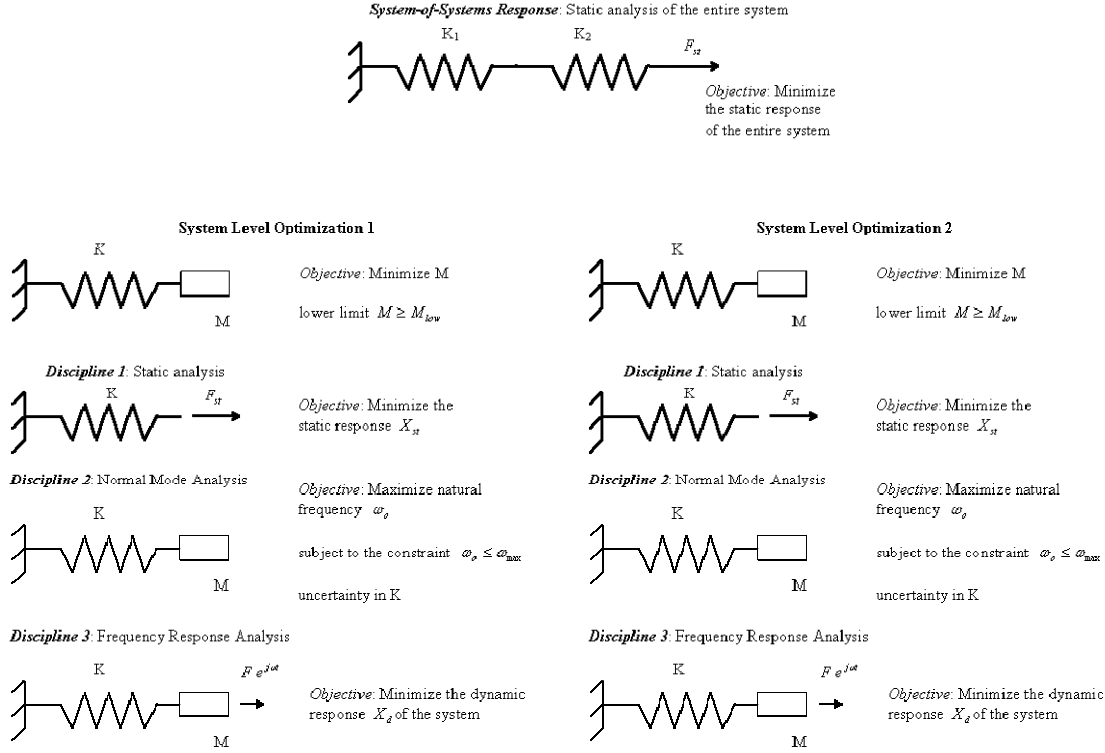
Figure 7. Outline of the SS-MDO-U process for the two single degree of freedom systems

V. Case Study

A representative case study associated with the simultaneous design of two ships is also analyzed in order to demonstrate the feasibility of applying the SS-MDO-U development in Naval applications of interest (such as the Seabasing concept). The case study accomplishes the following objectives:

(i)   Demonstrates how multiple ships can be analyzed jointly by considering each ship as a separate system with multiple disciplines and how the operational requirements for interaction between the two ships are considered at the very top system-of-systems level.

(ii)   Naval architecture disciplines are integrated in the SS-MDO-U process.

(iii)   Utilizes response surface Gaussian type metamodels for replacing time consuming analyses and for eliminating user intervention during the optimization process.

(iv)   Demonstrates how high fidelity simulations (i.e. UNDEX) can be replaced by FRM within the SS-MDO-U environment.

(v)   Identifies how uncertainty can influence the solution.

(vi)   Demonstrates how complex interactions among disciplines or among systems can be taken into account during the SS-MDO-U process.

The case study is generic but yet representative of a multiple ship design concept (like the seabasing), and demonstrates the feasibility of the SS-MDO-U concept. Figure 1 presents the overall set up of the optimization statement. Detailed information is provided next.

V.1. System-of-Systems Level

A HLS (indicated by subscript "1") and a TS (indicated by subscript "2") are considered to operate at the configuration presented in Figure 8. The heading of the waves considered during the simulation is also presented in Figure 8. A zero speed and a sea state 4 are considered in the simulation.
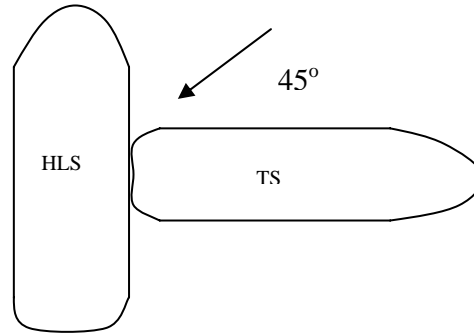
Figure 8. Two ship system

A cumulative measurable representing the relative motion between the two ships comprises the objective function at the system-of-systems level.

$$\left|Heave_1\right| + \left|Heave_2\right| + \left|B_1 * Roll_1\right| + \left|L_2 * Pitch_2\right| \tag{23}$$

where B indicates the beam of the ship, and L indicates the length. All the design variables of the two system level optimization statements are considered as design variables at the system-of-systems level (15 design variables). Constraints representative of the TC method are automatically defined as part of the optimization statement according to Equations (4). The objective function of Equation (23) is also augmented to include the limits of the TC constraints as part of the objective function according to Equation (3).

V.2. System Level Optimization for HLS

A cost function from the "Principles of Economics" notes from the UM comprises the objective function at the system level:

$$\text{minimize} \left(\frac{L_1 \, B_1 \, D_1}{100}\right)^{0.8} \tag{24}$$

Constraints representative of the TC method are automatically defined according to Equations (1). The objective function of Equation (24) is augmented to include the limits of the TC constraints according to Equation (1).

V.2.1. Seakeeping Discipline for HLS

The roll performance of the HLS comprises the objective function. Minimizing the roll of the HLS assists the objective function at the system-of-systems level. Constraints are imposed on the heave and pitch responses:

$$Heave_1 \leq 0.17, \quad Pitch_1 \leq 0.155 \tag{25}$$

All heave, pitch, and roll responses for both ships have been non-dimensionalized with 2m, 2° and 5°, respectively. The values for the 1/10 percentile of highest values are considered throughout the simulations for the seakeeping responses. The SPP code from UM is utilized for the seakeeping calculations. The SPP code is based on empirical seakeeping models and does not account for the influence between the two ships when evaluating the seakeeping response. A flow chart of the input and the output structure of the SPP is presented in Figure 9.
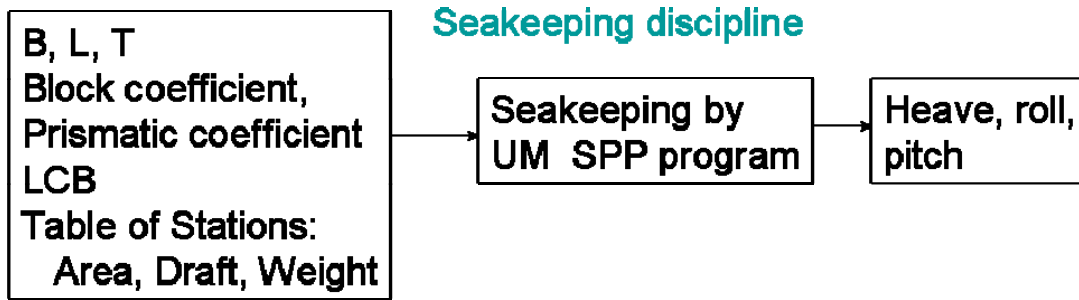
Figure 9. Input and output data structure for SPP code

Gaussian type metamodels are developed for each one of the three seakeeping responses of interest using the Kriging Method (Equations (7) – (17)). This eliminates the requirement of using the SPP code itself during the iterative optimization process. This elimination is necessary because the SPP requires from a user to interactively adjust certain parameters, thus making the direct utilization of the SPP impossible during an automated iterative process. Instead, a limited number of seakeeping analyses are performed off-line at a limited number of sample points selected by an optimal symmetric latin hypercube algorithm. The results at the sample points are used for creating the response surfaces.

The roll, heave, and pitch responses at the sample points are employed for constructing a separate response surface for each parameter. The beam, length, draft, and wave height comprise the varying parameters when developing the response surfaces. Figure 10 depicts the three response surfaces developed for the HLS when the draft and the wave height acquire their mean values and only the beam and the length are allowed to vary (this is done in order to visualize the response surfaces).
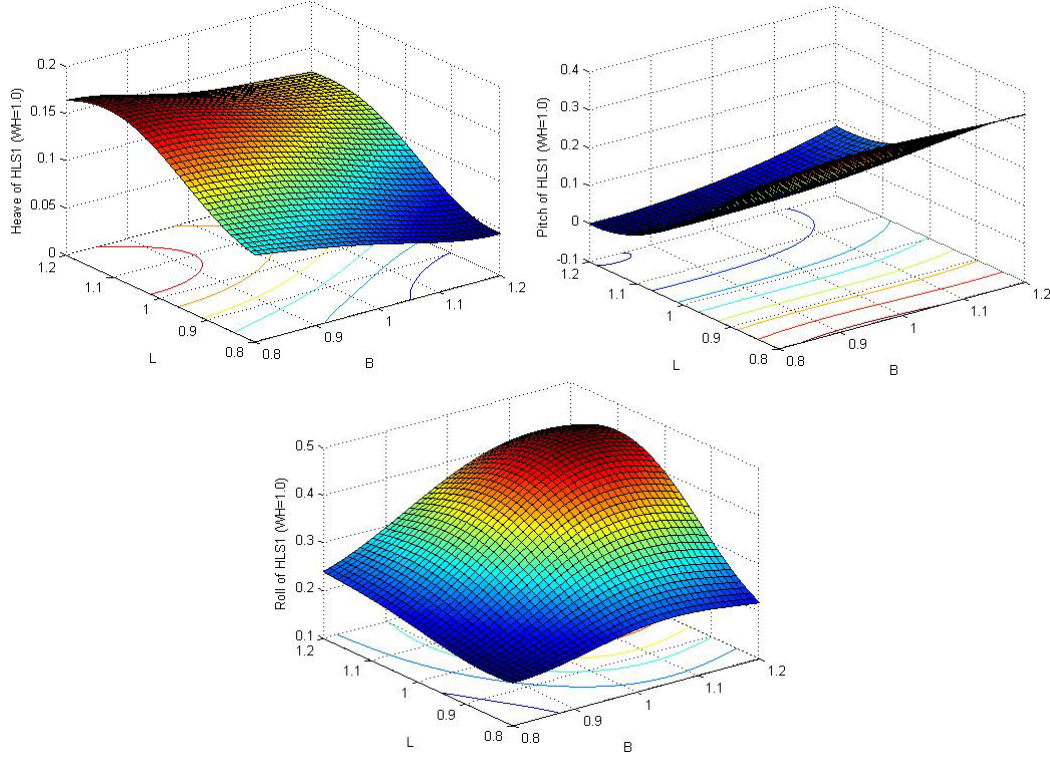
Figure 10. Response surfaces for heave, pitch, and roll of the HLS (keeping draft and wave height at their mean values for plotting purposes)

The beam and the length of the HLS comprise the design variables. However, the draft of the ship is adjusted to reflect the changes in the weight of the ship through the changes imposed by the structural strength discipline level analysis, and by changing the beam and the length of the ship. The beam and the length are allowed to vary between 120% and 80% of their original values.

V.2.2 Static Discipline for HLS
   A static analysis is considered for the HLS and the weight is minimized by retaining the SM above a prescribed value. Figure 11 presents how the mid-ship section of a typical HLS is idealized for this analysis.
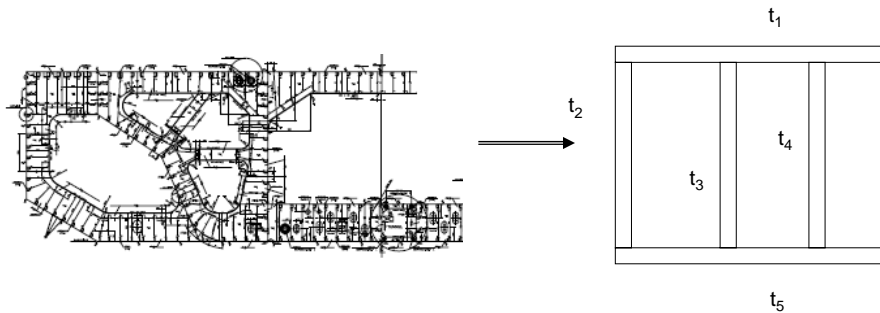
Figure 11. Simplified mid-ship section used in static analysis

The five thicknesses depicted in Figure 11 comprise the five design variables of this optimization and they are allowed to vary between 130% and 70% of their original values. The weight is

considered as a function of the five design variables but it also depends on the beam, length, and depth of the HLS. Thus, the outcome of the seakeeping discipline influences the static analysis discipline since the beam and the length are determined from the seakeeping analysis. A constraint is imposed on the SM based on the maximum possible dimensions which can be acquired by the ship. The requirement is for SM not to be smaller than 227.15.

V.3. System Level Optimization for the TS

The system design for the TS has been set up in an identical manner with the HLS. The main differences are in the dimensions and the hull form characteristics of the two ships and that an UNDEX simulation is performed instead of a static analysis. The latter is done in order to demonstrate the utilization of FRM instead of high fidelity simulations in the SS-MDO-U process. At the system level a cost function along with the limits of the TC constraints comprise the objective function similar to Equations (1) and (24). The TC constraints are defined according to Equations (1).

V.3.1 Seakeeping Discipline for TS

The seakeeping analysis is similar to the one performed for the HLS. The only difference is that the pitch is minimized in the objective function while constraints are imposed on heave and roll:

$$Heave_2 \leq 0.343$$

$$Roll_2 \leq 0.3$$

(26)

The same non-dimensionalization has been imposed to the heave, pitch, and roll responses of the TS as in the HLS case. Response surfaces are created and utilized in the optimization process similar to the HLS seakeeping analysis. Figure 12 presents the response surfaces for the TS.
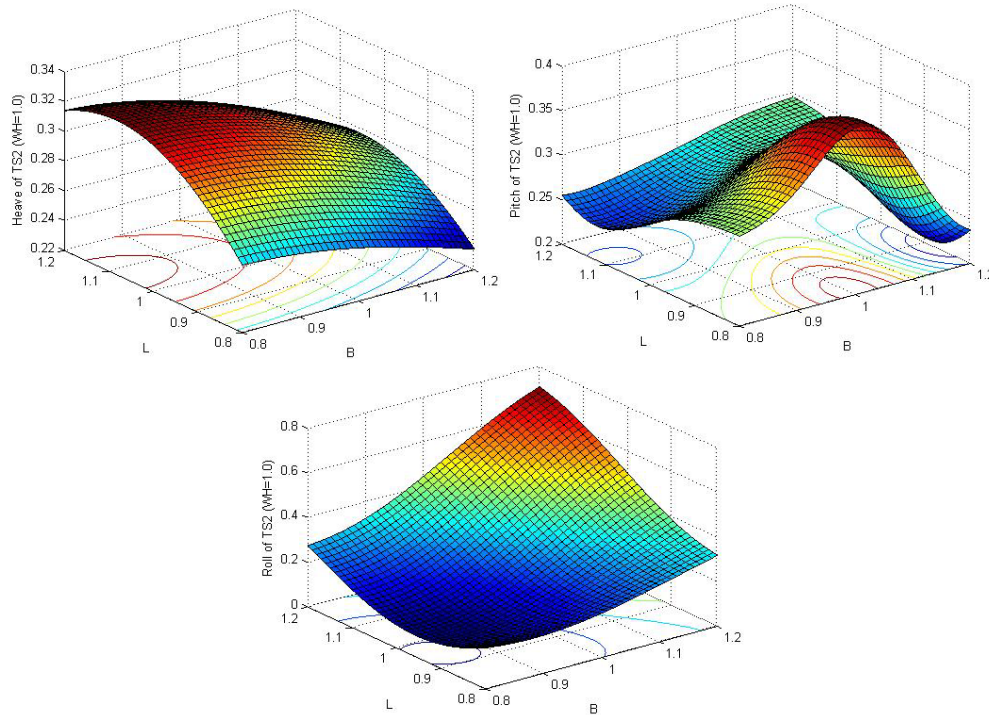
Figure 12. Response surfaces for heave, pitch, and roll of the TS (keeping draft and wave height at their mean values in this set of results)

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The beam and the length comprise the design variables but the draft is also adjusted (similar to the HLS seakeeping analysis) since the weight of the ship changes from the UNDEX consideration and from the changes in beam and length. The complete response surfaces (with varying draft and the wave height) are utilized during the optimization process.

V.3.2. UNDEX Discipline for TS

A generic ABAQUS FEA model (Figure 13) is utilized for performing UNDEX simulations for the TS. The FEA model for the ship is a simplistic one since the primary objective is to demonstrate how high fidelity simulations can be included in the SS-MDO-U process and developing a comprehensive FEA ship model is far beyond the scope of this project. In this work the ship is represented as a shell structure with modified modulus of elasticity in order to exhibit similar modal dynamic characteristics with an actual ship. The ship structure is divided into three sections and the density and modulus of elasticity of each section comprise the six design variables. Both the ship structure and the surrounding fluid are included in the FEA model. Infinite finite elements are employed for terminating the fluid elements and for introducing the non-reflective boundary conditions.
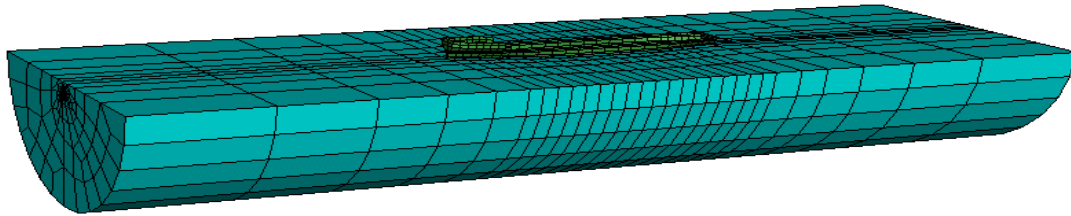
Figure 13. FEA model for TS utilized in UNDEX simulations using ABAQUS

FRM are developed based on a complete set of simulations performed at a limited set of sample points using Equations (18) – (22). A single ABAQUS simulation requires approximately 45 minutes. Therefore, performing multiple high fidelity simulations within an iterative optimization process is impractical. Instead, the FRM are constructed and utilized. The six design variables and the magnitude of the charge comprise the varying parameters when developing the FRM. The latter allows introducing the influence of the changes in the beam and the length of the TS to the loads applied on the structure from the explosion. The changes in the beam and the length originate from the seakeeping analysis. FRM are generated for the time histories of the Von Misses stress at three different representative monitoring locations at the keel. Figure 14 presents a representative correlation between the FRM and an actual ABAQUS simulation for a combination of the seven varying variables that was not utilized for generating the FRM. The results demonstrate a reliable prediction from the FRM. It is particularly important to notice that the ability of MES to produce FRM is unique and tailored to time dependent responses. The objective of this discipline optimization is to minimize the weight of the ship while constraining the maximum encountered stresses below the prescribed limit of 250MPa.
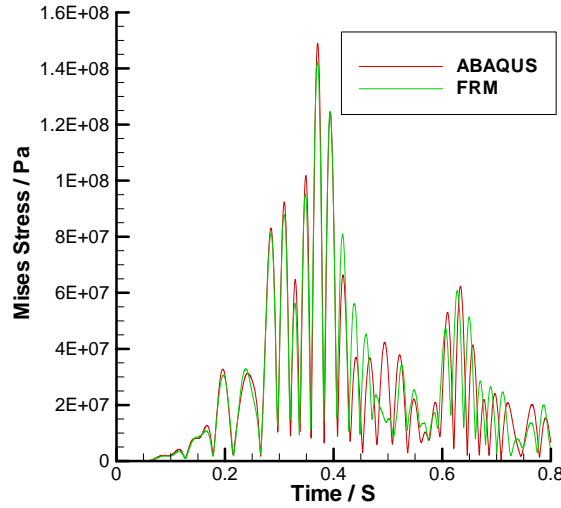
Figure 14. Representative correlation between FRM and actual UNDEX simulations for the time history of the Von Misses stress

V.4. Interactions among Disciplines and Interactions among Systems

The two disciplines for the HLS analysis are interacting with each other. The weight of the ship which changes during the static optimization influences the seakeeping optimization. At the same time the beam and the length of the ship influence the results of the static analysis since they participate in the definition of the objective function of the static analysis. In a similar manner the two disciplines for the TS analysis are interdependent. The weight of the ship which changes in the UNDEX simulation affects the draft in the seakeeping analysis. At the same time the beam and the length of the ship which are determined by the seakeeping simulations affect the magnitude of the load utilized in the UNDEX simulation.

The two ship systems are coupled through the definition of the objective function at the system-of-systems level. The TC method through the additional TC constraints and through the augmentations imposed to the objective functions organize the multiple optimization problems which are being solved and co-ordinate the changes introduced in the design variables at the various levels. In addition, the structured organization of the SS-MDO-U process that the TC method facilitates allows to account for the interaction between the multiple disciplines in a structured manner.

V.5. Including Uncertainty in the SS-MDO-U Case Study

In this case study two separate system of systems analyses are performed. One without considering any uncertainty and a second one that considers uncertainty in the wave height defined in the seakeeping analyses. The uncertainty influences the values of the heave, pitch, and roll, and therefore has the potential of violating certain constraints in the seakeeping disciplines. By making the constraints probabilistic instead of deterministic using Equations (6) ensures that the constraints are satisfied even in the presence of uncertainty. The seakeeping constraints become more conservative when the single loop RBDO algorithm is activated and uncertainty is considered in the SS-MDO-U analysis. The wave height comprises one of the variables when the response surfaces are created for the heave, roll, and pitch. In order to

demonstrate the effect of the wave height to the seakeeping responses the response surfaces for the pitch of the HLS are presented in Figure 15 for three different wave heights.
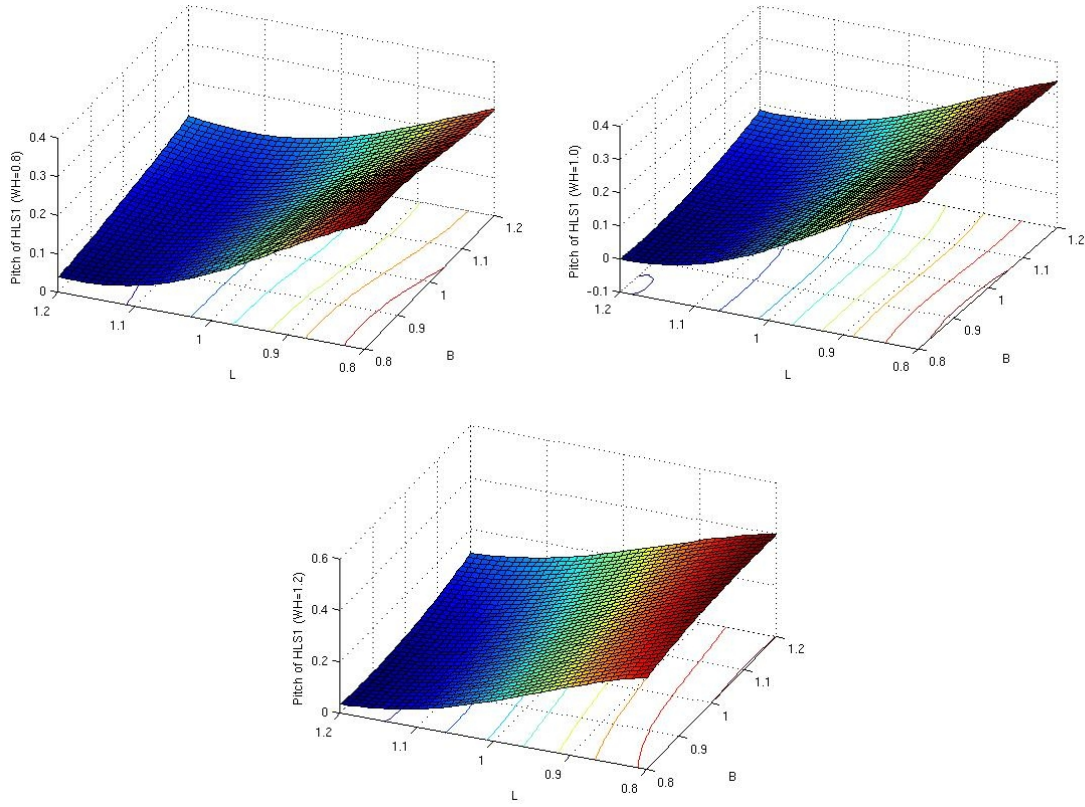
Figure 15. Response surfaces for the pitch response of the HLS for three different wave heights (80% of original value, original value, and 120% of the original value, respectively)

V.6. Summary of Results from Case Study

Table 2 summarizes the results from the two analyses without and with uncertainty. The values for the constraints within the parentheses contain the values that the constraints acquire in the presence of uncertainty. Subscript "1" indicates the HLS and subscript "2" indicates the TS. The thickness variables for the HLS are depicted in Figure 11. Subscripts "1" – "3" in the modulus of elasticity E and the density ρ of the TS represent the three different sections that the TS has been divided in. All heave, pitch, and roll responses for both ships have been non-dimensionalized with 2m, 2° and 5°, respectively. The values for the 1/10 percentile of highest values are considered throughout the simulations for the seakeeping responses and for any objective functions or constraints that contain information from the seakeeping analyses. All non-dimensional variables are indicated as such in Table 2.

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	Initial configuration	SS-MDO-U optimum without uncertainty	SS-MDO-U optimum with uncertainty
B_1	50m	0.8 ND (w.r.t. B_1)	0.801 ND (w.r.t. B_1)
L_1	180m	0.9783 ND (w.r.t. L_1)	1.003 ND (w.r.t. L_1)
t_1	0.3875m	1.1713 ND (w.r.t. t_1)	1.1713 ND (w.r.t. t_1)
t_2	0.2825m	1.1713 ND (w.r.t. t_2)	1.1713 ND (w.r.t. t_2)
t_3	0.2805m	0.7 ND (w.r.t. t_3)	0.7 ND (w.r.t. t_3)
t_4	0.28m	0.7 ND (w.r.t. t_4)	0.7 ND (w.r.t. t_4)
t_5	0.335m	0.7 ND (w.r.t. t_5)	0.7 ND (w.r.t. t_5)
B_2	25.4m	0.8 ND (w.r.t. B_2)	0.8589 ND (w.r.t. B_2)
L_2	175m	0.8 ND (w.r.t. L_2)	0.8 ND (w.r.t. L_2)
E_1	3.0E+14 Pa	1.0 ND (w.r.t. E_1)	1.0 ND (w.r.t. E_1)
E_2	3.0E+14 Pa	1.0 ND (w.r.t. E_3)	1.0 ND (w.r.t. E_3)
E_3	3.0E+14 Pa	1.0 ND (w.r.t. E_3)	1.0 ND (w.r.t. E_3)
ρ_1	7,800kg/m^3	0.702 ND (w.r.t. ρ_1)	0.702 ND (w.r.t. ρ_1)
ρ_2	7,800kg/m^3	0.7 ND (w.r.t. ρ_2)	0.7 ND (w.r.t. ρ_2)
ρ_3	7,800kg/m^3	0.7 ND (w.r.t. ρ_3)	0.7 ND (w.r.t. ρ_3)
System-of Systems Objective Function, ND	1	0.7417	0.7677
HLS system level Objective Function, ND Ship Cost	1	0.8221	0.8387
HLS Seakeeping Discipline Objective Function, Roll, ND	0.382	0.2057	0.2113
HLS Seakeeping Discipline, Constraint on Heave ≤ 0.17, ND	0.132	0.1458 (0.1524)	0.1545 (0.1588)
HLS Seakeeping Discipline, Constraint on Pitch ≤ 0.155, ND	0.117	0.1399 (0.1815)	0.1118 (0.155)
HLS Static Discipline, Objective function, Weight, ND	1	0.826	0.8469
HLS Static Discipline, Constraint on SM ≤ 227.15	252	227.8	228.2
TS System Level Objective Function, ND Ship Cost	1	0.6997	0.7407
TS Seakeeping Discipline Objective Function, Pitch, ND	0.317	0.3009	0.3147
TS Seakeeping Discipline, Constraint on Heave ≤ 0.343, ND	0.3054	0.265 (0.3075)	0.2638 (0.306)
TS Seakeeping Discipline, Constraint on Roll ≤ 0.3, ND	0.2274	0.184 (0.2752)	0.1702 (0.2809)
TS UNDEX Discipline, Objective Function, Weight, ND	1	0.4484	0.4810
TS UNDEX Discipline, Maximum Von Misses stress in Section $1 \leq 250$	108.68	72.6	76.76
TS UNDEX Discipline, Maximum Von Misses stress in Section $2 \leq 250$	207.13	104.74	112.69
TS UNDEX Discipline, Maximum Von Misses stress in Section $3 \leq 250$	83.37	37.84	40.606

Table 2. Summary of Results from Case Study; SS-MDO-U analyses of two ships; without uncertainty and with uncertainty

The main observations from the two SS-MDO-U analyses are:

(i)   There is a significant improvement (~25%) in the system-of-systems objective function that represents the maximum relative motion between the two ships.

(ii)   There is a significant reduction (~20% and ~30%) in the cost associated with both ships at each one of the two system levels objective functions.

(iii)   Due to the dependency of the cost functions on the main dimensions of the ships, the beam and the length of both ships is reduced as much as the lower bounds of the design variables and as much the seakeeping constraints allow. In addition, the requirement for minimizing the relative motion between the two ships at the system-of-systems level influences the reduction of the beam and length variables.

(iv)   The constraint on the pitch of the HLS is violated in the presence of uncertainty from the deterministic optimum configuration. When uncertainty is included in the SS-MDO-U process the optimal solution is adjusted and the constraint is satisfied even in the presence of uncertainty.

(v)   The variables for the modulus of elasticity of the TS do not change from their original values in the UNDEX discipline. That makes sense because the constraints (which are influenced by the E) do not become active and the objective function does not depend on E, therefore there is no reason for inducing any changes. The densities are associated with the weight of the ship and they are pushed all the way to their lower bound since the stress constraints remain inactive. The reason for the latter is the reduction in the total pressure load from the UNDEX due to the reduced footprint of the TS which is created from the reduction in beam and length.

Overall the SS-MDO-U process operates as expected. It leads to improved designs at the system-of-systems level and at each system level. It also accounts properly for the interactions among the multiple systems and among the multiple disciplines within each system.

   Figure 16 presents on the response surfaces of the heave, pitch, and roll of the HLS (under constant draft and wave height for visualization purposes, in reality these two variables change as well) the starting point, an intermediate point, and the final points for the optimized configurations without and with uncertainty. In the response surface for the pitch it can be observed how the optimal point under uncertainty has been adjusted to a lower pitch response in order to satisfy the constraint in the presence of uncertainty. The two other responses increase by small amounts in order to accommodate this change, but still remain within the feasible limits.
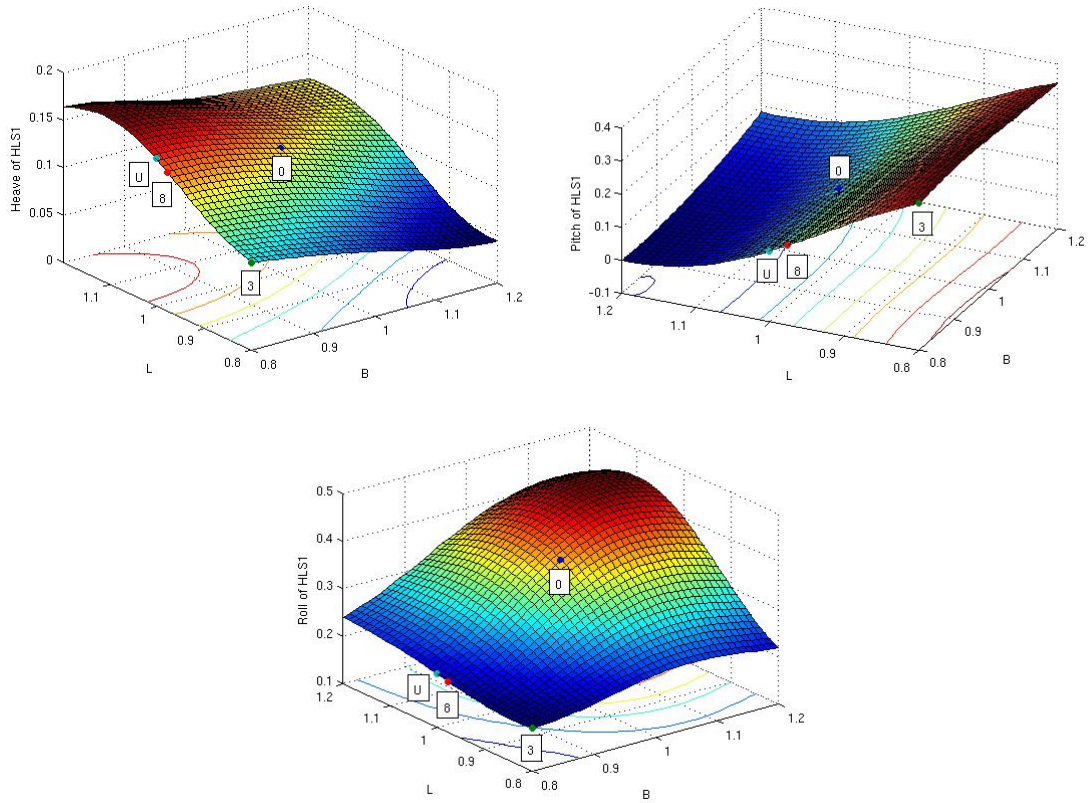
Figure 16. Tracing the starting Point "0", intermediate point "3", deterministic optimum "8", and probabilistic optimum "U" in the seakeeping response surfaces of the HLS (the response surfaces are presented under constant draft and wave height for visualization purposes, during the optimization these two variables vary as well)

VI. Contributions of this Project
 The following are the new contributions of the completed Phase I project:
- Developed a general purpose SS-MDO-U code.
- Introduced response surface and FRM capabilities in the SS-MDO-U development.
- Demonstrated through a simple analysis that the SS-MDO-U process operates as expected.
- Demonstrated through a generic but relevant to Naval interests case study that:
    (i)   Interactions among multiple ship systems can be accounted properly.
    (ii)   Naval Architecture disciplines can be included in the SS-MDO-U.
    (iii) High fidelity simulations can be utilized in the SS-MDO-U process.
    (iv)   Interactions among disciplines for a particular ship are accounted properly within each system analysis.
    (v)   Interaction and compatibility among systems is coordinated at the system-of-systems level.
    (vi)   Physical attributes driven by performance requirements can be linked to cost objectives within the SS-MDO-U process, thus optimizing performance while reducing the cost.

(vii) Accounting for uncertainties during the SS-MDO-U simulations introduces conservatism in the constraints and ensures that the optimal point will satisfy the constraints even in the presence of uncertainty.

The TC approach dictates the structure of the SS-MDO-U system. Overall, the TC divides a very complex and complicated optimization challenge into a group of smaller and systematically interconnected optimization analyses. It allows to process the results from each system and each discipline separately and thus identifies the critical factors in the decision making process. The TC approach provides:

(i) a symmetric organization to the discipline level optimizations within each system analysis
(ii) a systematic flow of data among the disciplines of each system
(iii) a system of organized links between multiple systems at the system-of-systems level analysis
(iv) a capability to drive the optimal point to a more conservative solution in order to avoid violation of all constraints in the presence of uncertainty

VII. Potential New Developments and Market Potential

The Phase I STTR effort established the core competency of the SS-MDO-U capability. It also demonstrated the feasibility of the new technology through a case study. The core value of the SS-MDO-U is that it accounts for interactions and common physical attributes between multiple disciplines within a system, it integrates multiple systems for a system-of-systems level analysis and it relates physical characteristics to cost. It provides the enabling technology for dividing a very complex and interconnected design problem into a network of multiple smaller and systematically linked optimization problems, thus making the solution feasible. At the same time it allows to identify the critical points and parameters to the overall decision making process. During the Phase II effort the following Tasks will be pursued:

(i) Utilize game theory in order to convert desired operational and performance characteristics into mathematical statements that can be employed in the mathematical articulation of an optimization statement.
(ii) Develop an algorithm which identifies the feasibility of the starting selection with respect to all of the constraints. Since multiple designs and multiple disciplines within each design are considered during the system of systems optimization process, it is important to ensure that the initial starting selection of design variables resides within the feasible region for all constraints from all disciplines and all individual ship platforms.
(iii) Develop an algorithm that adjusts the final optimum in order to ensure that all the constraints from the lower level disciplines and systems are satisfied. Since optimizations are performed at multiple levels of the SS-MDO-U process, the adjustment of the design variables is performed at multiple levels. The development under this task will ensure that all of the constraints of the various level optimizations will be satisfied at the end of the SS-MDO-U process.
(iv) Identify the most important variables which have been encountered during the optimization process. This capability will provide information to the user about the nature of the optimization that is been solved and the relative importance of the various variables utilized in the process.

(v)   Create a user interface which will guide the user in creating the data file for the optimization process and facilitate the visualization of the results. In order to increase the utilization of the new product a "wizard" based interface will guide the user in the required definition of the design variables, objective functions, and constraints required for the mathematical articulation of the SS-MDO-U optimization process.

(vi)   Enhance the library of optimization algorithms which can be employed for guiding the multiple optimization processes. This will provide additional flexibility in definition of the optimization statements.

(vii) Develop a capability for handling discrete design variables.

(viii) Perform a case study representative of the Seabasing concept.

The market potential is extensively discussed in the submitted STTR Transition Plan.

Appendix A.
SS-MDO-U data files for the case study analyses without uncertainty and with uncertainty

$SS-MDO-U Analysis without Uncertainty
$
$$
$ Define system of systems level optimization
*BEGIN_SOS
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of system of systems level optimization - SOS_Main.m
*BEGIN_MAIN
$
*DESIGNVARIABLE
$ name   initial-value   lower-bound   upper-bound   nondimensionalization-facor   Standard-deviation

name	initial-value	lower-bound	upper-bound	nondimensionalization-facor
B1	50.0	0.8*50.0	1.2*50.0	50.0
L1	180.0	0.8*180.0	1.2*180.0	180.0
t1	0.3875	0.7*0.3875	1.3*0.3875	0.3875
t2	0.2825	0.7*0.2825	1.3*0.2825	0.2825
t3	0.2805	0.7*0.2805	1.3*0.2805	0.2805
t4	0.2800	0.7*0.2800	1.3*0.2800	0.2800
t5	0.335	0.7*0.3350	1.3*0.3350	0.3350
B2	25.4	0.8*25.400	1.2*25.400	25.400
L2	175.0	0.8*175.00	1.2*175.00	175.00
E1	3.0e14	0.7*3.0e14	1.3*3.0e14	3.0e14
E2	3.0e14	0.7*3.0e14	1.3*3.0e14	3.0e14
E3	3.0e14	0.7*3.0e14	1.3*3.0e14	3.0e14
rho1	7800.0	0.7*7800.0	1.3*7800.0	7800.0
rho2	7800.0	0.7*7800.0	1.3*7800.0	7800.0
rho3	7800.0	0.7*7800.0	1.3*7800.0	7800.0

$
*OPTIMIZATION_OPTION
$ option   value
  Display   iter
  Diagnostics   off
  LargeScale   off
  MaxFunEvals   300
  DiffMinChange 0.005
  DiffMaxChange 0.10
  TolCon   1.0e-2
  TolX   1.0e-2
  TolFun   1.0e-2
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of system of systems level optimization -
SOS_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name   value
  WH1   2.2
  Weight0 (50.0/2*(0.3875+0.2825)+12*(0.2805+0.2800+0.3350))*180.0
  Weight1 (B1/2*(t1+t2)+12*(t3+t4+t5))*L1
  DispH   61239.9
  Disp1   DispH*(Weight1-Weight0)/Weight0
  TD1   Disp1/B1/L1/1025.0/0.723+9.0
  WH2   2.2
  A1   3698.82*B2*L2/25.4/175.0
  A2   5449.17*B2*L2/25.4/175.0
  A3   3848.20*B2*L2/25.4/175.0
  Mass0   7800.0*3698.82+7800.0*5449.17+7800.0*3848.20
  Mass1   rho1*A1+rho2*A2+rho3*A3

```
    DispT    22929.1
    Disp2    DispT*(Mass1-Mass0)/Mass0
    TD2      Disp2/B2/L2/1025.0/0.723+9.0
    r1       [B1/50.0, L1/180.0, TD1/9.0, WH1/2.2]
    r2       [B2/25.4, L2/175.0, TD2/9.0, WH2/2.2]
    r2rl     [B2/25.4, L2/175.0, TD2/9.0, WH2/2.2]
*UTILITYPROGRAM_FUNCTION
$ function-call
   Heave1 = seakeeping('FRM1h.mat', r1);
   Roll1 = seakeeping('FRM1r.mat', r1);
   Heave2 = seakeeping('FRM2h.mat', r2);
   Pitch2 = seakeeping('FRM2p.mat', r2rl);
$
*PARAMETER
$ name         value
   Broll1       B1*Roll1/57.3
   Lpitch2      L2*Pitch2/57.3
   zhpr         abs(Broll1)+abs(Lpitch2)+abs(Heave1)+abs(Heave2)
$
*OBJVAL
$ objective-function-value
zhpr
$
*END_OBJ
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of system of systems level optimization -
SOS_Constraint.m
*BEGIN_CONSTRAINT
$
*SYSTEM_OPTIMIZATION
$ system-name
   HLS1
$
*SYSTEM_OPTIMIZATION
$ system-name
   TS2
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_SOS
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$ Define System HLS1
*BEGIN_SYSTEM
$ system-name
   HLS1
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define top level optimization
*BEGIN_TOP
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of top level optimization - HLS1_Top_Main.m
*BEGIN_MAIN
$
*LINKINGVARIABLE
$ name    lower-bounds    upper-bounds
   B1         0.9             1.1
   L1         0.9             1.1
   t1         0.8             1.2
```

```
      t2          0.8              1.2
      t3          0.8              1.2
      t4          0.8              1.2
      t5          0.8              1.2
$
*OPTIMIZATION_OPTION
$ option          value
      Display      off
      Diagnostics  off
      LargeScale   off
      MaxFunEvals  50
      DiffMinChange  0.005
      DiffMaxChange  0.10
      TolCon       1.0e-2
      TolX         1.0e-2
      TolFun       1.0e-2
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of top level optimization - HLS1_Top_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name          value
      Dh          12.0
$
*OBJVAL
$ objective-function-value
      (L1*B1*Dh/100.0)^0.8/267.1
$
*END_OBJ
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of top level optimization - HLS1_Top_Constraint.m
*BEGIN_CONSTRAINT
$
*DISCIPLINE_OPTIMIZATION
$ discpline-number
      D1
$
*DISCIPLINE_OPTIMIZATION
$ discipline-number
      D2
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_TOP
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
*BEGIN_DISCIPLINE
$ discipline-number
      D1
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of discipline D1 - HLS1_D1_Main.m
*BEGIN_MAIN
$
*LINKINGVARIABLE
```

```
$ name      lower-bounds    upper-bounds
  B1          0.9             1.1
  L1          0.9             1.1
  t1          1.0             1.0
  t2          1.0             1.0
  t3          1.0             1.0
  t4          1.0             1.0
  t5          1.0             1.0
$
*OPTIMIZATION_OPTION
$ option        value
  Display        off
  Diagnostics    off
  LargeScale     off
  MaxFunEvals    50
  DiffMinChange  0.005
  DiffMaxChange  0.10
  TolCon         1.0e-2
  TolX           1.0e-2
  TolFun         1.0e-2
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of discipline D1 optimization - HLS1_D1_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name     value
  WH1        2.2
  Weight0  (50.0/2*(0.3875+0.2825)+12*(0.2805+0.2800+0.3350))*180.0
  Weight1  (B1/2*(t1+t2)+12*(t3+t4+t5))*L1
  DispH     61239.9
  Disp1     DispH*(Weight1-Weight0)/Weight0
  TD1       Disp1/B1/L1/1025.0/0.723+9.0
  r        [B1/50.0, L1/180.0,TD1/9.0,WH1/2.2]
$
*UTILITYPROGRAM_FUNCTION
$ function-call
  Roll = seakeeping('FRM1r.mat', r)
$
*OBJVAL
$ objective-function-value
  Roll/0.381
$
*END_OBJ
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of discipline D1 optimization - HLS1_D1_Constraint.m
*BEGIN_CONSTRAINT
$
*PARAMETER
$ name      value
  WH1        2.2
  Weight0  (50.0/2*(0.3875+0.2825)+12*(0.2805+0.2800+0.3350))*180.0
  Weight1  (B1/2*(t1+t2)+12*(t3+t4+t5))*L1
  DispH     61239.9
  Disp1     DispH*(Weight1-Weight0)/Weight0
  TD1       Disp1/B1/L1/1025.0/0.723+9.0
  r        [B1/50.0, L1/180.0,TD1/9.0,WH1/2.2]
```

```
$
*UTILITYPROGRAM_FUNCTION
$ function-call
  Heave = seakeeping('FRM1h.mat', r)
$
*UTILITYPROGRAM_FUNCTION
$ function-call
  Pitch = seakeeping('FRM1p.mat', r)
$
*CVAL
$ inequality-constraint-function-value
  Heave-0.170
  Pitch-0.155
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_DISCIPLINE
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
*BEGIN_DISCIPLINE
$ discipline-number
  D2
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of discipline D2 - HLS1_D2_Main.m
*BEGIN_MAIN
$
$
*LINKINGVARIABLE
$ name    lower-bounds    upper-bounds
  B1        1.0             1.0
  L1        1.0             1.0
  t1        0.8             1.2
  t2        0.8             1.2
  t3        0.8             1.2
  t4        0.8             1.2
  t5        0.8             1.2
$
$
*OPTIMIZATION_OPTION
$ option        value
  Display       off
  Diagnostics   off
  LargeScale    off
  MaxFunEvals   50
  DiffMinChange 0.005
  DiffMaxChange 0.10
  TolCon        1.0e-2
  TolX          1.0e-2
  TolFun        1.0e-2
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of discipline D2 optimization - HLS1_D2_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name          value
  weight1       (B1/2*(t1+t2)+12*(t3+t4+t5))*L1
```

```
$
*OBJVAL
$ objective-function-value
  weight1/4950
$
*END_OBJ
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of discipline D2 optimization - HLS1_D2_Constraint.m
*BEGIN_CONSTRAINT
$
*UTILITYPROGRAM_FUNCTION
$ function-call
  Secmod=SM(t1,t2,t3,t4,t5,B1)
$
*CVAL
$ inequality-constraint-function-value
  227.15-Secmod
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_DISCIPLINE
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
*END_SYSTEM
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$ Define System TS2
*BEGIN_SYSTEM
$ system-name
  TS2
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define top level optimization
*BEGIN_TOP
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of top level optimization - TS2_Top_Main.m
*BEGIN_MAIN
$
*LINKINGVARIABLE
$ name    lower-bounds    upper-bounds
  B2         0.90            1.10
  L2         0.90            1.10
  E1         0.8             1.2
  E2         0.8             1.2
  E3         0.8             1.2
  rho1       0.8             1.2
  rho2       0.8             1.2
  rho3       0.8             1.2
$
*OPTIMIZATION_OPTION
$ option         value
  Display        off
  Diagnostics    off
  LargeScale     off
  MaxFunEvals    50
  DiffMinChange  0.005
  DiffMaxChange  0.10
```

```
    TolCon          1.0e-2
    TolX            1.0e-2
    TolFun          1.0e-2
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of top level optimization - TS2_Top_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name          value
    Dt            16.0
$
*OBJVAL
$ objective-function-value
    (L2*B2*Dt/100.0)^0.8/191.25
$
*END_OBJ
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of top level optimization - TS2_Top_Constraint.m
*BEGIN_CONSTRAINT
$
*DISCIPLINE_OPTIMIZATION
$ displine-number
    D1
$
*DISCIPLINE_OPTIMIZATION
$ discipline-number
    D2
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_TOP
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
*BEGIN_DISCIPLINE
$ discipline-number
    D1
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of discipline D1 - TS2_D1_Main.m
*BEGIN_MAIN
$
*LINKINGVARIABLE
$ name    lower-bounds    upper-bounds
    B2        0.9             1.1
    L2        0.9             1.1
    rho1      1.0             1.0
    rho2      1.0             1.0
    rho3      1.0             1.0
$
*OPTIMIZATION_OPTION
$ option          value
    Display         off
    Diagnostics     off
    LargeScale      off
    MaxFunEvals     50
```

```
   DiffMinChange  0.005
   DiffMaxChange  0.10
   TolCon         1.0e-2
   TolX           1.0e-2
   TolFun         1.0e-2
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of discipline D1 optimization - TS2_D1_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name    value
   WH2       2.2
   A1        3698.82*B2*L2/25.4/175.0
   A2        5449.17*B2*L2/25.4/175.0
   A3        3848.20*B2*L2/25.4/175.0
   Mass0     7800.0*3698.82+7800.0*5449.17+7800.0*3848.20
   Mass1     rho1*A1+rho2*A2+rho3*A3
   DispT     22929.1
   Disp2     DispT*(Mass1-Mass0)/Mass0
   TD2       Disp2/B2/L2/1025.0/0.723+9.0
   r         [B2/25.4, L2/175.0,TD2/9.0,WH2/2.2]
$
*UTILITYPROGRAM_FUNCTION
$ function-call
   Tpitch = seakeeping('FRM2p.mat', r)
$
*OBJVAL
$ objective-function-value
   Tpitch/0.111
$
*END_OBJ
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of discipline D1 optimization - TS2_D1_Constraint.m
*BEGIN_CONSTRAINT
$
*PARAMETER
$ name    value
   WH2       2.2
   A1        3698.82*B2*L2/25.4/175.0
   A2        5449.17*B2*L2/25.4/175.0
   A3        3848.20*B2*L2/25.4/175.0
   Mass0     7800.0*3698.82+7800.0*5449.17+7800.0*3848.20
   Mass1     rho1*A1+rho2*A2+rho3*A3
   DispT     22929.1
   Disp2     DispT*(Mass1-Mass0)/Mass0
   TD2       Disp2/B2/L2/1025.0/0.723+9.0
   r         [B2/25.40, L2/175.0,TD2/9.0,WH2/2.2]
$
*UTILITYPROGRAM_FUNCTION
$ function-call
   Theave = seakeeping('FRM2h.mat', r)
$
*UTILITYPROGRAM_FUNCTION
$ function-call
   Troll = seakeeping('FRM2r.mat', r)
$
```

case_study[1].txt

```
*CVAL
$ inequality-constraint-function-value
   Theave-0.343
   Troll-0.300
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_DISCIPLINE
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
*BEGIN_DISCIPLINE
$ discipline-number
   D2
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of discipline D2 - TS2_D2_Main.m
*BEGIN_MAIN
$
*LINKINGVARIABLE
$ name    lower-bounds    upper-bounds
   B2       1.0             1.0
   L2       1.0             1.0
   E1       0.8             1.2
   E2       0.8             1.2
   E3       0.8             1.2
   rho1     0.8             1.2
   rho2     0.8             1.2
   rho3     0.8             1.2
$
*OPTIMIZATION_OPTION
$ option        value
   Display       off
   Diagnostics   off
   LargeScale    off
   MaxFunEvals   50
   DiffMinChange 0.005
   DiffMaxChange 0.10
   TolCon        1.0e-2
   TolX          1.0e-2
   TolFun        1.0e-2
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of discipline D2 optimization - TS2_D2_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name        value
   A1          3698.82*B2*L2/25.4/175.0
   A2          5449.17*B2*L2/25.4/175.0
   A3          3848.20*B2*L2/25.4/175.0
   Weight2     (rho1*A1+rho2*A2+rho3*A3)*0.0762
$
*OBJVAL
$ objective-function-value
   Weight2/7.7244e4
$
*END_OBJ
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of discipline D2 optimization - TS2_D2_Constraint.m
*BEGIN_CONSTRAINT
$
*PARAMETER
$ name    value
   pres   0.2*B2*L2/25.4/175.0
   r      [E1/3.0e14, rho1/7800.0,E2/3.0e14, rho2/7800.0,E3/3.0e14,
rho3/7800.0,pres/0.2]
$
*UTILITYPROGRAM_FUNCTION
$ function-call
  MaxMS1 = MaxStress('FRM1.mat', r);
  MaxMS2 = MaxStress('FRM2.mat', r);
  MaxMS3 = MaxStress('FRM3.mat', r)
$
*CVAL
$ inequality-constraint-function-value
  MaxMS1-250.0e6
  MaxMS2-250.0e6
  MaxMS3-250.0e6
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_DISCIPLINE
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
*END_SYSTEM
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
```

```
$ SS-MDO-U analysis with Uncertainty
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$ Define system of systems level optimization
*BEGIN_SOS
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of system of systems level optimization - SOS_Main.m
*BEGIN_MAIN
$
*DESIGNVARIABLE
$ name   initial-value   lower-bound    upper-bound     nondimensionalization-facor
Standard-deviation
    B1       50.0          0.8*50.0        1.2*50.0              50.0
    L1       180.0         0.8*180.0       1.2*180.0             180.0
    t1       0.3875        0.7*0.3875      1.3*0.3875            0.3875
    t2       0.2825        0.7*0.2825      1.3*0.2825            0.2825
    t3       0.2805        0.7*0.2805      1.3*0.2805            0.2805
    t4       0.2800        0.7*0.2800      1.3*0.2800            0.2800
    t5       0.335         0.7*0.3350      1.3*0.3350            0.3350
    B2       25.4          0.8*25.400      1.2*25.400            25.400
    L2       175.0         0.8*175.00      1.2*175.00            175.00
    E1       3.0e14        0.7*3.0e14      1.3*3.0e14            3.0e14
    E2       3.0e14        0.7*3.0e14      1.3*3.0e14            3.0e14
    E3       3.0e14        0.7*3.0e14      1.3*3.0e14            3.0e14
    rho1     7800.0        0.7*7800.0      1.3*7800.0            7800.0
    rho2     7800.0        0.7*7800.0      1.3*7800.0            7800.0
    rho3     7800.0        0.7*7800.0      1.3*7800.0            7800.0
$
*OPTIMIZATION_OPTION
$ option            value
  Display           iter
  Diagnostics       off
  LargeScale        off
  MaxFunEvals       300
  DiffMinChange     0.005
  DiffMaxChange     0.10
  TolCon            1.0e-2
  TolX              1.0e-2
  TolFun            1.0e-2
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of system of systems level optimization -
SOS_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name          value
   WH1          2.2
   Weight0      (50.0/2*(0.3875+0.2825)+12*(0.2805+0.2800+0.3350))*180.0
   Weight1      (B1/2*(t1+t2)+12*(t3+t4+t5))*L1
   DispH        61239.9
   Disp1        DispH*(Weight1-Weight0)/Weight0
   TD1          Disp1/B1/L1/1025.0/0.723+9.0
   WH2          2.2
   A1           3698.82*B2*L2/25.4/175.0
   A2           5449.17*B2*L2/25.4/175.0
   A3           3848.20*B2*L2/25.4/175.0
   Mass0        7800.0*3698.82+7800.0*5449.17+7800.0*3848.20
   Mass1        rho10*A1+rho20*A2+rho30*A3
```

```
      DispT     22929.1
      Disp2     DispT*(Mass1-Mass0)/Mass0
      TD2       Disp2/B2/L2/1025.0/0.723+9.0
      r1        [B1/50.0, L1/180.0, TD1/9.0, WH1/2.2]
      r2        [B2/25.4, L2/175.0, TD2/9.0, WH2/2.2]
      r2rl      [B2/25.4, L2/175.0, TD2/9.0, WH2/2.2]
*UTILITYPROGRAM_FUNCTION
$ function-call
   Heave1 = seakeeping('FRM1h.mat', r1);
   Roll1 = seakeeping('FRM1r.mat', r1);
   Heave2 = seakeeping('FRM2h.mat', r2);
   Pitch2 = seakeeping('FRM2p.mat', r2rl);
$
*PARAMETER
$ name         value
   Broll1       B1*Roll1/57.3
   Lpitch2      L2*Pitch2/57.3
   zhpr         abs(Broll1)+abs(Lpitch2)+abs(Heave1)+abs(Heave2)
$
*OBJVAL
$ objective-function-value
zhpr
$
*END_OBJ
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of system of systems level optimization -
SOS_Constraint.m
*BEGIN_CONSTRAINT
$
*SYSTEM_OPTIMIZATION
$ system-name
   HLS1
$
*SYSTEM_OPTIMIZATION
$ system-name
   TS2
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_SOS
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$ Define System HLS1
*BEGIN_SYSTEM
$ system-name
   HLS1
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define top level optimization
*BEGIN_TOP
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of top level optimization - HLS1_Top_Main.m
*BEGIN_MAIN
$
*LINKINGVARIABLE
$ name   lower-bounds    upper-bounds
   B1        0.9             1.1
   L1        0.9             1.1
   t1        0.8             1.2
```

```
   t2          0.8              1.2
   t3          0.8              1.2
   t4          0.8              1.2
   t5          0.8              1.2
$
*OPTIMIZATION_OPTION
$ option          value
   Display        off
   Diagnostics    off
   LargeScale     off
   MaxFunEvals    50
   DiffMinChange  0.005
   DiffMaxChange  0.10
   TolCon         1.0e-2
   TolX           1.0e-2
   TolFun         1.0e-2
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of top level optimization - HLS1_Top_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name          value
   Dh            12.0
$
*OBJVAL
$ objective-function-value
   (L1*B1*Dh/100.0)^0.8/267.1
$
*END_OBJ
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of top level optimization - HLS1_Top_Constraint.m
*BEGIN_CONSTRAINT
$
*DISCIPLINE_OPTIMIZATION
$ displine-number
   D1
$
*DISCIPLINE_OPTIMIZATION
$ discipline-number
   D2
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_TOP
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
*BEGIN_DISCIPLINE
$ discipline-number
   D1
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of discipline D1 - HLS1_D1_Main.m
*BEGIN_MAIN
$
*LINKINGVARIABLE
```

```
$ name      lower-bounds    upper-bounds
  B1          0.9             1.1
  L1          0.9             1.1
  t1          1.0             1.0
  t2          1.0             1.0
  t3          1.0             1.0
  t4          1.0             1.0
  t5          1.0             1.0
$
*OPTIMIZATION_OPTION
$ option        value
  Display        off
  Diagnostics    off
  LargeScale     off
  MaxFunEvals    50
  DiffMinChange  0.005
  DiffMaxChange  0.10
  TolCon         1.0e-2
  TolX           1.0e-2
  TolFun         1.0e-2
$
*PARAMETER_U
$ name    mean    standard-deviation
  W1      2.2          0.2136
$
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of discipline D1 optimization - HLS1_D1_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name     value
   WH1        2.2
  Weight0  (50.0/2*(0.3875+0.2825)+12*(0.2805+0.2800+0.3350))*180.0
  Weight1  (B1/2*(t1+t2)+12*(t3+t4+t5))*L1
  DispH    61239.9
  Disp1    DispH*(Weight1-Weight0)/Weight0
   TD1      Disp1/B1/L1/1025.0/0.723+9.0
   r       [B1/50.0, L1/180.0,TD1/9.0,WH1/2.2]
$
*UTILITYPROGRAM_FUNCTION
$ function-call
  Roll = seakeeping('FRM1r.mat', r)
$
*OBJVAL
$ objective-function-value
  Roll/0.381
$
*END_OBJ
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of discipline D1 optimization - HLS1_D1_Constraint.m
*BEGIN_CONSTRAINT
$
*PARAMETER
$ name      value
$   WH1        2.2
  Weight0  (50.0/2*(0.3875+0.2825)+12*(0.2805+0.2800+0.3350))*180.0
```

```
   Weight1   (B1/2*(t1+t2)+12*(t3+t4+t5))*L1
   DispH     61239.9
   Disp1     DispH*(Weight1-Weight0)/Weight0
    TD1       Disp1/B1/L1/1025.0/0.723+9.0
$
*CVAL_U
$ inequality-constraint-function-value  target-reliability-index
   HLS1D1Cons1(B1,L1,TD1,W1)                           2.06
$ v&p_u1   v&p_u2   v&p_u3   v&p_u4   v&p_u5   v&p_u6   v&p_u7   v&p_u8
   W1
$ diff1    diff2    diff3    diff4    diff5    diff6    diff7    diff8
   0.01*W1
$
*CVAL_U
$ inequality-constraint-function-value  target-reliability-index
   HLS1D1Cons2(B1,L1,TD1,W1)                           2.06
$ v&p_u1   v&p_u2   v&p_u3   v&p_u4   v&p_u5   v&p_u6   v&p_u7   v&p_u8
   W1
$ diff1    diff2    diff3    diff4    diff5    diff6    diff7    diff8
   0.01*W1
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_DISCIPLINE
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
*BEGIN_DISCIPLINE
$ discipline-number
   D2
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of discipline D2 - HLS1_D2_Main.m
*BEGIN_MAIN
$
$
*LINKINGVARIABLE
$ name    lower-bounds    upper-bounds
   B1       1.0             1.0
   L1       1.0             1.0
   t1       0.8             1.2
   t2       0.8             1.2
   t3       0.8             1.2
   t4       0.8             1.2
   t5       0.8             1.2
$
$
*OPTIMIZATION_OPTION
$ option          value
   Display        off
   Diagnostics    off
   LargeScale     off
   MaxFunEvals    50
   DiffMinChange  0.005
   DiffMaxChange  0.10
   TolCon         1.0e-2
   TolX           1.0e-2
   TolFun         1.0e-2
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
```

```
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of discipline D2 optimization - HLS1_D2_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name          value
  weight1       (B1/2*(t1+t2)+12*(t3+t4+t5))*L1
$
*OBJVAL
$ objective-function-value
  weight1/49.50
$
*END_OBJ
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of discipline D2 optimization - HLS1_D2_Constraint.m
*BEGIN_CONSTRAINT
$
*UTILITYPROGRAM_FUNCTION
$ function-call
  Secmod=SM(t1,t2,t3,t4,t5,B1)
$
*CVAL
$ inequality-constraint-function-value
  227.15-Secmod
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_DISCIPLINE
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
*END_SYSTEM
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$ Define System TS2
*BEGIN_SYSTEM
$ system-name
  TS2
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define top level optimization
*BEGIN_TOP
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of top level optimization - TS2_Top_Main.m
*BEGIN_MAIN
$
*LINKINGVARIABLE
$ name    lower-bounds    upper-bounds
  B2        0.90            1.10
  L2        0.90            1.10
  E1        0.8             1.2
  E2        0.8             1.2
  E3        0.8             1.2
  rho1      0.8             1.2
  rho2      0.8             1.2
  rho3      0.8             1.2
$
*OPTIMIZATION_OPTION
```

```
$ option       value
  Display       off
  Diagnostics   off
  LargeScale    off
  MaxFunEvals   50
  DiffMinChange 0.005
  DiffMaxChange 0.10
  TolCon        1.0e-2
  TolX          1.0e-2
  TolFun        1.0e-2
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of top level optimization - TS2_Top_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name        value
  Dt          16.0
$
*OBJVAL
$ objective-function-value
  (L2*B2*Dt/100.0)^0.8/191.25
$
*END_OBJ
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of top level optimization - TS2_Top_Constraint.m
*BEGIN_CONSTRAINT
$
*DISCIPLINE_OPTIMIZATION
$ displine-number
  D1
$
*DISCIPLINE_OPTIMIZATION
$ discipline-number
  D2
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_TOP
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
*BEGIN_DISCIPLINE
$ discipline-number
  D1
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define main part of discipline D1 - TS2_D1_Main.m
*BEGIN_MAIN
$
*LINKINGVARIABLE
$ name    lower-bounds    upper-bounds
  B2        0.9             1.1
  L2        0.9             1.1
  rho1      1.0             1.0
  rho2      1.0             1.0
  rho3      1.0             1.0
```

```
$
*OPTIMIZATION_OPTION
$ option        value
  Display       off
  Diagnostics   off
  LargeScale    off
  MaxFunEvals   50
  DiffMinChange  0.005
  DiffMaxChange  0.10
  TolCon        1.0e-2
  TolX          1.0e-2
  TolFun        1.0e-2
$
*PARAMETER_U
$ name    mean    standard-deviation
  W2      2.2          0.2136
$
*END_MAIN
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define objective function of discipline D1 optimization - TS2_D1_Objective.m
*BEGIN_OBJ
$
*PARAMETER
$ name     value
   WH2       2.2
   A1        3698.82*B2*L2/25.4/175.0
   A2        5449.17*B2*L2/25.4/175.0
   A3        3848.20*B2*L2/25.4/175.0
   Mass0    7800.0*3698.82+7800.0*5449.17+7800.0*3848.20
   Mass1    rho10*A1+rho20*A2+rho30*A3
   DispT    22929.1
   Disp2    DispT*(Mass1-Mass0)/Mass0
   TD2      Disp2/B2/L2/1025.0/0.723+9.0
   r      [B2/25.4, L2/175.0,TD2/9.0,WH2/2.2]
$
*UTILITYPROGRAM_FUNCTION
$ function-call
  Tpitch = seakeeping('FRM2p.mat', r)
$
*OBJVAL
$ objective-function-value
  Tpitch/0.111
$
*END_OBJ
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of discipline D1 optimization - TS2_D1_Constraint.m
*BEGIN_CONSTRAINT
$
*PARAMETER
$ name     value
$   WH2       2.2
   A1        3698.82*B2*L2/25.4/175.0
   A2        5449.17*B2*L2/25.4/175.0
   A3        3848.20*B2*L2/25.4/175.0
   Mass0    7800.0*3698.82+7800.0*5449.17+7800.0*3848.20
   Mass1    rho10*A1+rho20*A2+rho30*A3
   DispT    22929.1
   Disp2    DispT*(Mass1-Mass0)/Mass0
```

    TD2   Disp2/B2/L2/1025.0/0.723+9.0
$
*CVAL_U
$ inequality-constraint-function-value target-reliability-index
   TS2D1Cons1(B2, L2, TD2, W2)   2.06
$ v&p_u1   v&p_u2   v&p_u3   v&p_u4   v&p_u5   v&p_u6   v&p_u7   v&p_u8
   W2
$ diff1   diff2   diff3   diff4   diff5   diff6   diff7   diff8
   0.01*W2
$
*CVAL_U
$ inequality-constraint-function-value target-reliability-index
   TS2D1Cons2(B2, L2, TD2, W2)   2.06
$ v&p_u1   v&p_u2   v&p_u3   v&p_u4   v&p_u5   v&p_u6   v&p_u7   v&p_u8
   W2
$ diff1   diff2   diff3   diff4   diff5   diff6   diff7   diff8
   0.01*W2
$
$
*END_CONSTRAINT
$%%%
$
*END_DISCIPLINE
$$$
$
$$$
*BEGIN_DISCIPLINE
$ discipline-number
   D2
$
$%%%
$ Define main part of discipline D2 - TS2_D2_Main.m
*BEGIN_MAIN
$
*LINKINGVARIABLE
$ name   lower-bounds   upper-bounds
   B2   1.0   1.0
   L2   1.0   1.0
   E1   0.8   1.2
   E2   0.8   1.2
   E3   0.8   1.2
   rho1   0.8   1.2
   rho2   0.8   1.2
   rho3   0.8   1.2
$
*OPTIMIZATION_OPTION
$ option   value
   Display   off
   Diagnostics   off
   LargeScale   off
   MaxFunEvals   50
   DiffMinChange 0.005
   DiffMaxChange 0.10
   TolCon   1.0e-2
   TolX   1.0e-2
   TolFun   1.0e-2
$
*END_MAIN
$%%%
$
$%%%
$ Define objective function of discipline D2 optimization - TS2_D2_Objective.m
*BEGIN_OBJ

```
$
*PARAMETER
$ name          value
    A1          3698.82*B2*L2/25.4/175.0
    A2          5449.17*B2*L2/25.4/175.0
    A3          3848.20*B2*L2/25.4/175.0
  Weight2       (rho1*A1+rho2*A2+rho3*A3)*0.0762
$
*OBJVAL
$ objective-function-value
  Weight2/7.7244e4
$
*END_OBJ
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$ Define constraint function of discipline D2 optimization - TS2_D2_Constraint.m
*BEGIN_CONSTRAINT
$
*PARAMETER
$ name    value
    pres   0.2*B2*L2/25.4/175.0
    r      [E1/3.0e14, rho1/7800.0,E2/3.0e14, rho2/7800.0,E3/3.0e14,
rho3/7800.0,pres/0.2]
$
*UTILITYPROGRAM_FUNCTION
$ function-call
  MaxMS1 = MaxStress('FRM1.mat', r);
  MaxMS2 = MaxStress('FRM2.mat', r);
  MaxMS3 = MaxStress('FRM3.mat', r)
$
*CVAL
$ inequality-constraint-function-value
  MaxMS1-250.0e6
  MaxMS2-250.0e6
  MaxMS3-250.0e6
$
*END_CONSTRAINT
$%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
$
*END_DISCIPLINE
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
*END_SYSTEM
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
```